Improvement of ICCG Convergence for Thin Elements in Magnetic Field Analyses Using Finite Element Method

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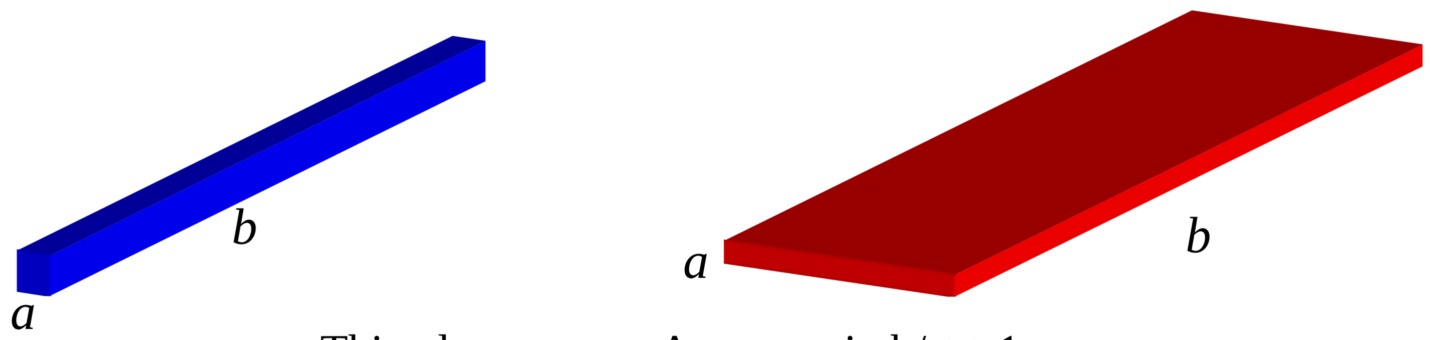
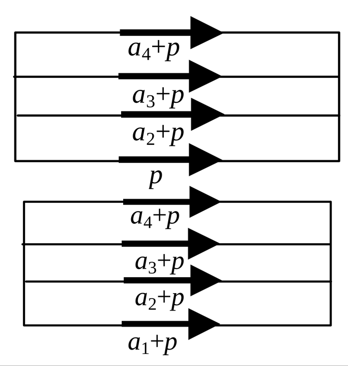
How do we overcome the degradation of ICCG using thin elements?



Thin elements Aspect ratio b/a>>1

- ●Special elements “Gap elements“ -- Fujiwara et al.(1996)
  Reconstruction of connections of solid to shell elements is necessary when it is analyzed from wide to thin gap in the same time.
- ●Using variables of potential difference
  -- Ueyama et al.(1990), Muramatsu et al.(2006)
- ●Proposed method using “Singularity Decomposition Technique”
  Nearly the same with the method by Muramatsu, but very easy to be implemented.



“Singularity Decomposition Technique”

	Slow converging components
■*A*-ϕ formulation in low frequency	Irrotational field components Add $\nabla\phi$
■Implicit Correction Multigrid Method	Low space-frequency components Add coarse mesh correction
■Proposed method for thin elements	Constant component between near parallel edges Add constant variables

●The addition of redundant variables improves ICCG characteristic.

Simple equation

$$\begin{bmatrix} 1 & -1+\varepsilon \\ -1+\varepsilon & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

$\varepsilon << 1.$

Nearly singular

Eigen values

$\lambda = \varepsilon,\ 2-\varepsilon$

Condition number

$\lambda_{max}/\lambda_{min} = (2-\varepsilon)/\varepsilon \approx 2/\varepsilon$

↓ Decomposition

$x_1 = y_1 + z,$ $x_2 = y_2 + z.$ $\quad \boldsymbol{x} = \boldsymbol{y} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} z$

↓ Corrected equation

$$\begin{bmatrix} 1 & -1+\varepsilon & \varepsilon \\ -1+\varepsilon & 1 & \varepsilon \\ \varepsilon & \varepsilon & 2\varepsilon \end{bmatrix}\begin{bmatrix} x_1 \\ y_2 \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_1+b_2 \end{bmatrix}$$

↓ Scaling

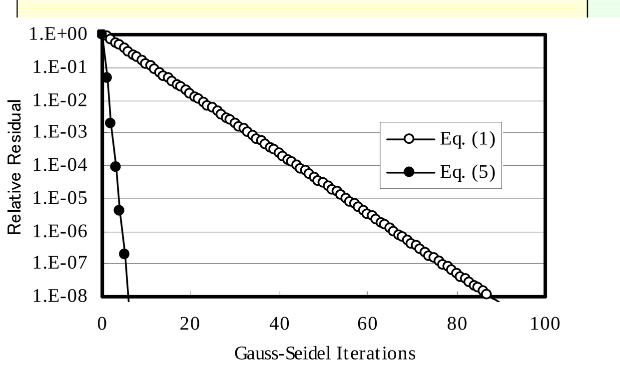
$$\begin{bmatrix} 1 & -1+\varepsilon & \sqrt{\varepsilon/2} \\ -1+\varepsilon & 1 & \sqrt{\varepsilon/2} \\ \sqrt{\varepsilon/2} & \sqrt{\varepsilon/2} & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_2 \\ z' \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ (b_1+b_2)/\sqrt{2\varepsilon} \end{bmatrix}, \quad z = \frac{z'}{\sqrt{2\varepsilon}}$$

Eigen values

$\lambda = 0,\ 1+\varepsilon,\ 2-\varepsilon$

Condition number

$\lambda_{max}/\lambda_{min} = (2-\varepsilon)/(1+\varepsilon) \approx 2$



By Gauss-Seidel Method

Formal

$\boldsymbol{Ax} = \boldsymbol{b}$

↓ Decomposition

$\boldsymbol{x} = \boldsymbol{y} + P\boldsymbol{z}$

$P = [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n]$

$\boldsymbol{v}_i$'s are eigen vectors of smallest eigen values or vectors with small norm $\boldsymbol{v}^t A \boldsymbol{v}$.

↓ Corrected equation

$$\begin{bmatrix} A & AP \\ P^t A & P^t AP \end{bmatrix}\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b} \\ P^t \boldsymbol{b} \end{bmatrix}$$

↓ Scaling

LDL decomposition

CG Solver

A-ϕ formulation

$\nabla\times\nu\nabla\times\boldsymbol{A} + j\sigma\omega\boldsymbol{A} = \boldsymbol{J}$

when σω is small .

↓ Decomposition

$\boldsymbol{A} = \boldsymbol{A}' + \nabla\phi$

$\nabla\times\nu\nabla\times(\nabla\phi) = 0$

$\sigma\omega(\nabla\phi)$ is small.

↓ Corrected equation

$\nabla\times\nu\nabla\times\boldsymbol{A}' + j\sigma\omega(\boldsymbol{A}' + \nabla\phi) = \boldsymbol{J}$

$-\nabla\cdot j\sigma\omega(\boldsymbol{A}' + \nabla\phi) = 0$

P: Incident matrix between edges and nodes.

↓

ICCG Solver

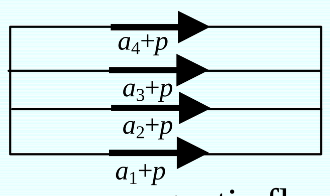
Implicit Correction Multigrid Method

P: Prolongation (Interpolation) matrix.

Low space-frequency (coarse mesh) components have small norms.

Proposed method for thin elements

Constant variable *p* for a group of parallel near edges.



Constant variable means no magnetic flux between parallel edges ⟶ Small norms

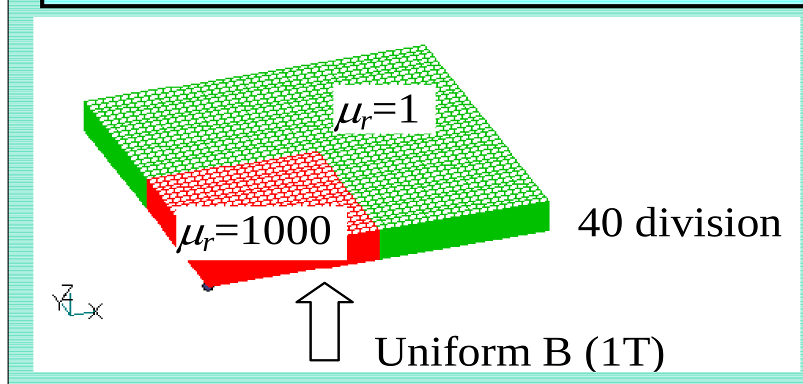
Conclusion

◆The *A*-ϕ formulation, Implicit Correction Multigrid Method and proposed method for thin elements have the same structure, which called as “Singular Decomposition Technique”.

◆The proposed method improve the ICCG convergence by 10 or more times faster than ordinary method when extremely thin elements are used.

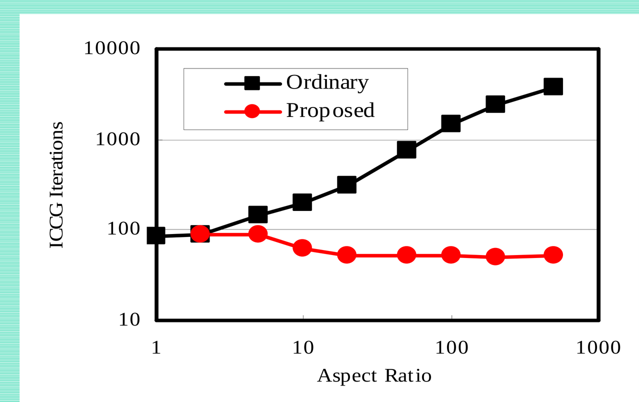
Numerical Tests

2-D Problem by 3-D Calculation



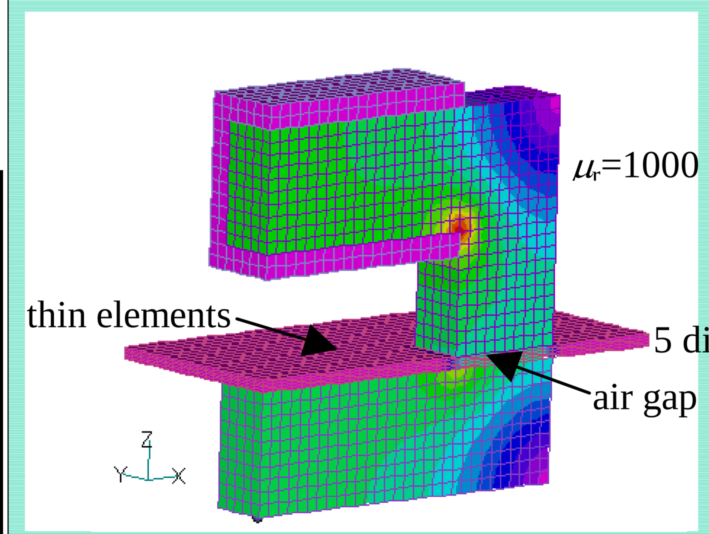
Analytic solution

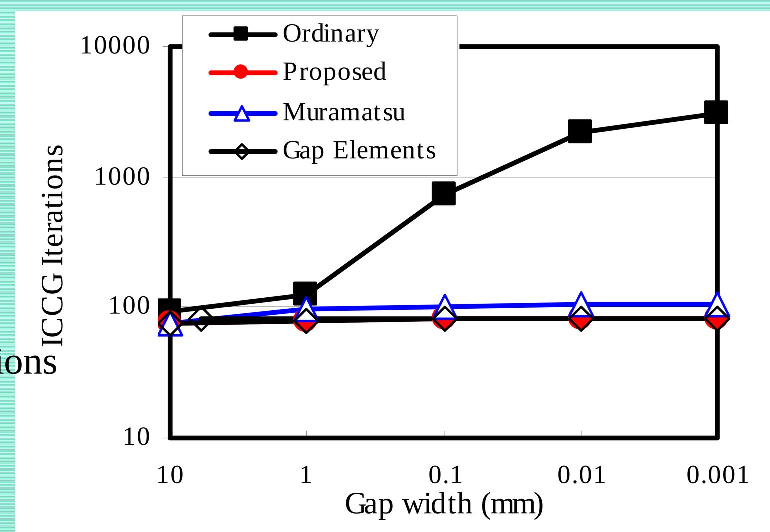
Bz :4/1.003 T in iron, 4/1003 T in air



Number of ICCG iterations v.s. element aspect ratio.

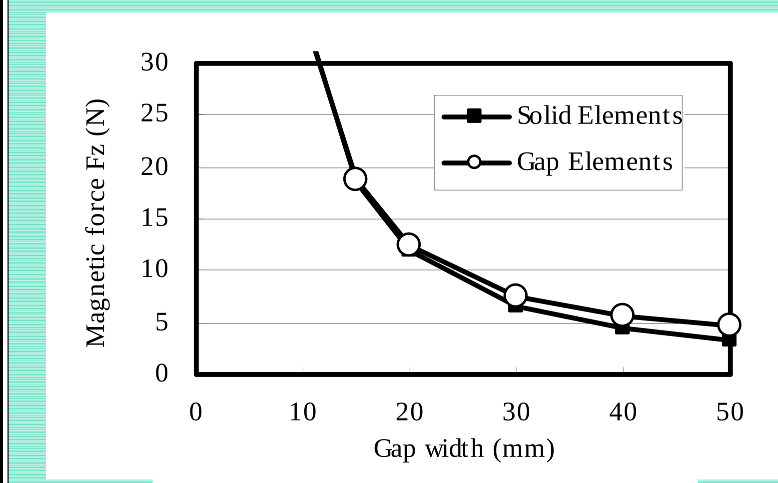
Problem with Air Gap in Magnetic Circuit



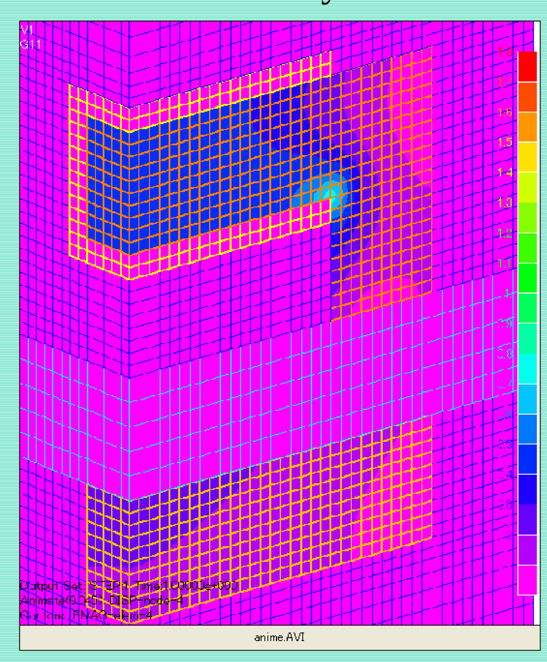


Number of ICCG iterations v.s. element aspect ratio.

Proposed method is slightly faster than the method by Muramatsu.

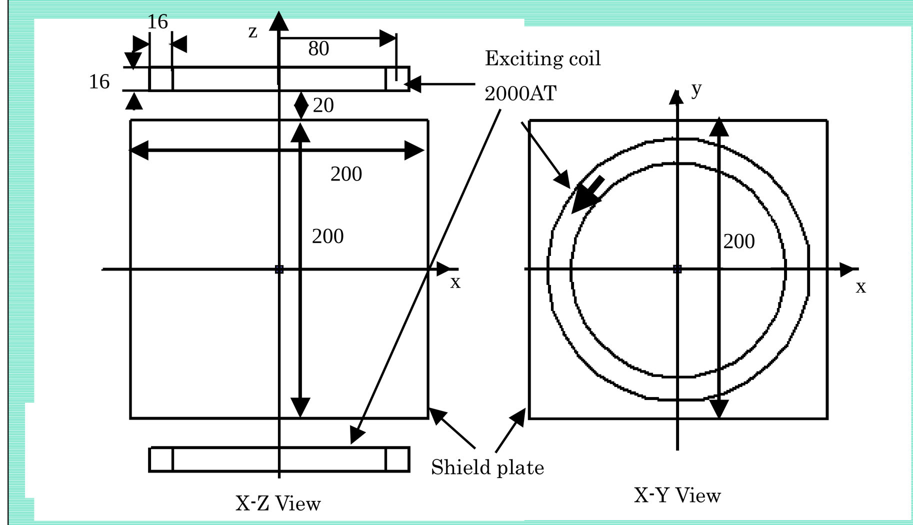


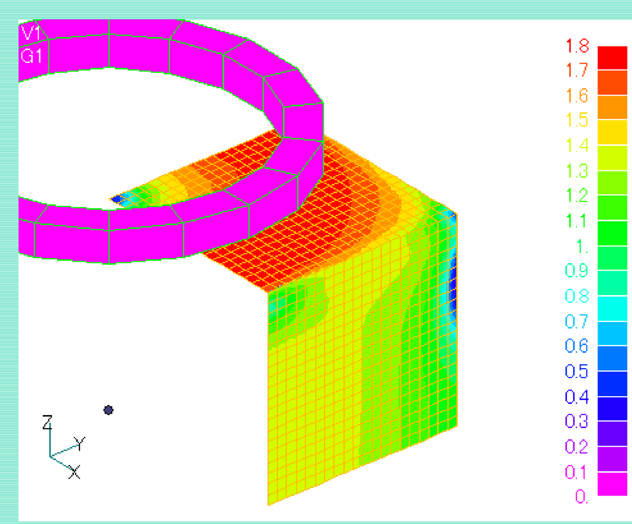
Magnetic force v.s. gap width



Gap element can simulate small gaps, but not large gaps.

Box Shield Model (Wall Thickness 0.01mm)





Division into 10 layers of 0.01mm wall

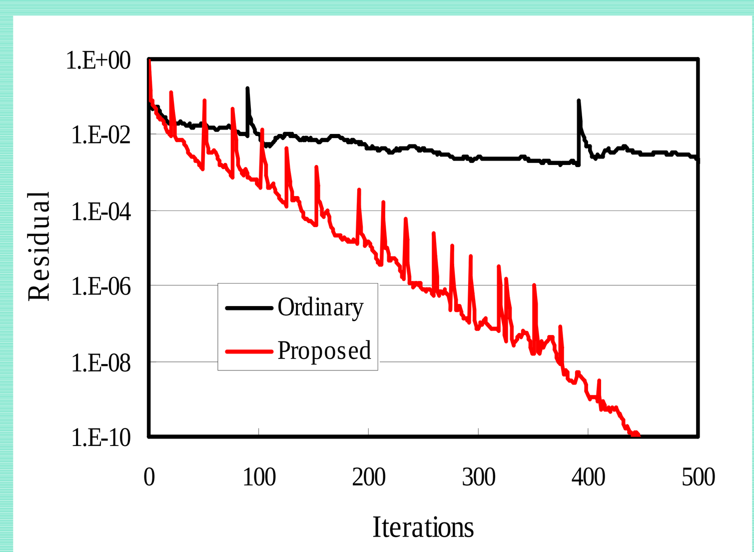
Maximum element aspect ratios are 5,000 in the wall and 20,000 in the air.

Table I. Calculation data for box shield model

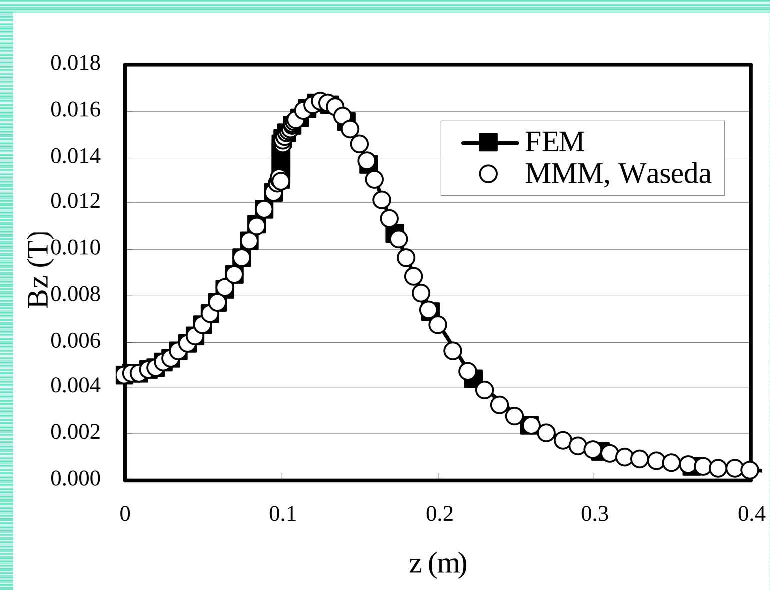
	Ordinary	Proposed
Number of elements	125,000	
Number of Nodes	132,651	
Number of Unknowns	365,050	374,370
Number of Non-zeros	6,016,122	7,606,586
N-R iterations	25	19
Total ICCG iterations	13410	546
CPU time (s)	2056.3	223.1

CPU time is only 11% !

●Results are the same between ordinary and proposed methods.



Convergence of Newton-Raphson and ICCG methods.



Bz v.s. z on z-axis.

Results by the proposed method agreed well with the result by Magnetic Moment Method by Takahashi.